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ESTIMATION OF RANGE AND DEPTH OF A SUBMERGED MOVING OBJECT BY USING NOISE CEPSTRUM[†]

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This paper attempts to come up with a method to measure the range and depth from the sound source to the receiver by analyzing radiated noise cepstrum of a moving vessel or torpedo. Two sets of formulas have been deduced in the process. The paper uses a moving object as an illustrative example. The normalized short-time spectrograph of the radiated noise from a submerged moving object is analyzed to establish an interference pattern. Through the analysis of cepstrum, a cepstrum graph with two bright lines is obtained. The bright lines represent the two time delay differences—that between the direct path and the sea surface reflection path and that between the direct path and the sea bottom reflection path. If both time delay differences are available at the same time, the first set of formulas can be used to calculate the range and depth of the sound source. If only one time delay difference is available, the second set of formulas can be used to measure the range of the sound source by supposing that the depth is already known. An experiment demonstrates that the range and depth calculated by the formulas conform to the on-site results.

1. INTRODUCTION

Cepstrum analysis has found wide applications in many fields. In reference [1], cepstrum processing was used to extract the difference between the two paths with a greater sensitivity than autocorrelation. In reference [2], cepstrum analysis of pulse intervals within sperm whale sonar clicks was used to estimate the whale's body length. In reference [3], cepstrum techniques were used to determine the source depth from the spectra of small explosions observed at long ranges. In reference [4], cepstrum techniques were used to estimate the inter-path time delay, and reduce the location error of passive geolocation for the case of multi-path propagation.

This paper makes an attempt to estimate the range and depth of a submerged moving sound source, for example, underwater propellers of a vessel, submarine and torpedo, by using cepstrum analysis. A submerged moving object may produce radiated noise, which has many features. These features are resulted either from the source of the sound, or the underwater acoustics channels through which the sound passes. The position, depth and moving parameters of a moving object may be acquired by analyzing the features. In many experiments conducted in shallow waters, the interferences between the sounds from the sea surface reflection path and the direct path could often be identified. Sometimes,

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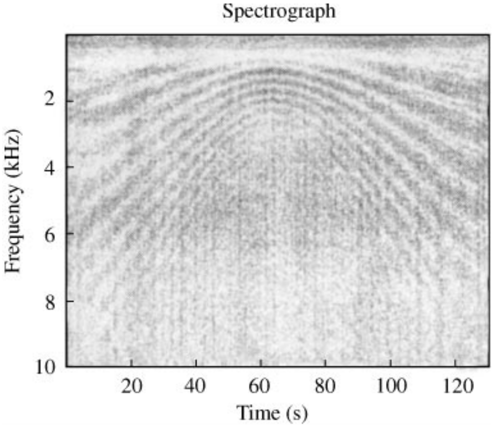

Figure 1. Spectrograph of a vessel radiated noise.

interferences between the sea bottom reflection path and the direct path could also be identified. Figure 1 represents a normalized short-time spectrograph of the radiated noise from a submerged moving object, in which an interference pattern showing an obvious hyperbola can be found. Based on this observation, this paper tries to develop a methodology of cepstrum analysis for estimating the range and depth of a moving sound source. By using such analysis in our experiment, we succeeded in estimating the range and depth of the moving sound source.

The most important advantage of the methodology introduced in this paper is that the range between a moving sound source and a receiver can be measured in passive ways by using only one single sensor. In underwater acoustics, echoes are usually employed to measure the range. Several transducers placed separately in an array have to be used to measure range in a passive way. For example, three transducers have to be positioned on a submarine, respectively, at the bow, the middle and the stern of the boat, to receive the radiated noise from the source. Then by comparing the time delay differences, the distance can be calculated. However, there are many conditions in which it would be impossible to place several transducers at certain distances, for example, in the case of a torpedo or a mine. Using a single sensor in a passive way to measure the range provides a meaningful solution to such cases.

In the following section, the paper would first deal with the formation of the interference pattern and the approach to analyze it.

2. ANALYSIS OF THE INTERFERENCE PATTERN

Figure 2 is the geometry of noise propagation from the surface reflection path, the bottom reflection path and the direct path. S is the sound source, F_1 is the receiving point. The ranges for the direct path and surface reflection path are R_d and $(R_{r1} + R_{r2})$ respectively. The range for the bottom reflection path is $(R'_{r1} + R'_{r2})$. H_t is the depth of the sound source and H_r is the depth of the receiving point. H_{sea} is the depth of the sea. The point of F_2 is the surface mirror image of the receiving point, while the point of F_3 is the sea bottom mirror image of the receiving point. The distance between the sound source and the surface mirror image is R_r, while that between the sound source and the sea bottom mirror images is R'_r.

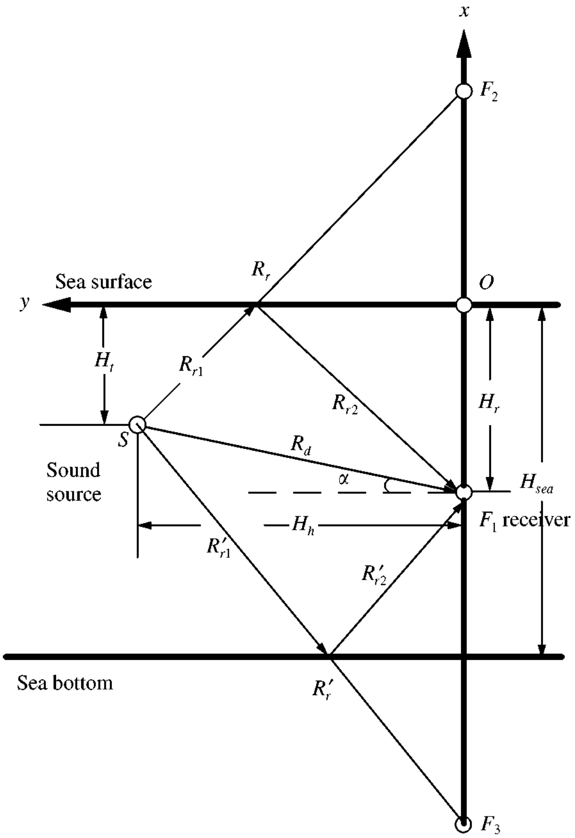

Figure 2. Geometry of sound paths.

2.1. ANALYSIS OF INTERFERENCE PATTERN THROUGH TWO PATHS (THE DIRECT PATH AND THE SURFACE REFLECTION PATH)

It is supposed that the situation is non-dispersive (i.e., the propagation velocity through each path is independent of frequency) and the noise $x(t)$ of the sound source is from a stationary (ergodic) random process $\{x(t)\}$, which will produce the signal $y(t)$ at the receiver

$$y(t) = H_1 x(t - \tau_d) + H_2 x(t - \tau_r), \tag{1}$$

where H_1 and H_2 are, respectively, uniform gain factor for the direct path and the surface reflection path. τ_d is the time delay of the direct path, τ_r is the time delay of the surface reflection path. τ_d is equal to R_d/C, C is the velocity of the sound. τ_r is equal to R_r/C. Reference [5] gives the autospectrum $G_{yy}(f)$ of $y(t)$ in equation (1), where f is frequency.

$$G_{yy}(f) = G_{xx}(f)\,[H_1^2 + H_2^2 + 2H_1H_2\cos 2\pi f(\tau_r - \tau_d)], \tag{2}$$

which is the sum of spectra through the two paths with a superimposed interference pattern at the following frequency:

$$\Delta f_{dr} = 1/(\tau_r - \tau_d). \tag{3}$$

2.2. ANALYSIS OF INTERFERENCE PATTERN THROUGH THREE PATHS

Now, let us consider the situation with three paths (the direct path, the sea surface reflection path and the sea bottom reflection path), wherein the signal $y(t)$ at the receiver becomes

$$y = H_1 x(t - \tau_d) + H_2 x(t - \tau_r) + H_3 x(t - \tau_r'), \tag{4}$$

where H_3 is a uniform gain factor for the sea bottom reflection path, and τ_r' is the time delay of the sea bottom reflection path. τ_r' is equal to R_r'/C. With the same arithmetic, the autospectrum $G_{yy}(f)$ of $y(t)$ in equation (4) is then

$$\begin{aligned} G_{yy}(f) = G_{xx}(f)\,[&H_1^2 + H_2^2 + H_3^2 + 2H_1H_2\cos 2\pi f(\tau_r - \tau_d) \\ &+ 2H_1H_3\cos 2\pi f(\tau_r' - \tau_d) + 2H_2H_3\cos 2\pi f(\tau_r' - \tau_r)], \end{aligned} \tag{5}$$

which is the sum of spectra through the three paths with a superimposed interference pattern including three components at the following frequencies:

$$\Delta f_{dr1} = 1/(\tau_r - \tau_d) = 1/\tau_1, \tag{6}$$

$$\Delta f_{dr2} = 1/(\tau_r' - \tau_d) = 1/\tau_2, \tag{7}$$

$$\Delta f_{dr3} = 1/(\tau_r' - \tau_r) = 1/\tau_3, \tag{8}$$

where τ_1 is the time delay difference between the direct path and the sea surface reflection path, τ_2 is the time delay difference between the direct path and the sea bottom reflection path, τ_3 is the time delay difference between the sea surface reflection path and the sea bottom reflection path.

2.3. ANALYSIS OF THE INTERFERENCE PATTERN OF THE MOVING SOUND SOURCE

The above analysis illustrates how the interference pattern of the moving object in Figure 1 is formed. The sound source came from the distance, crossed the receiving point and moved on away from the receiving point. The whole process lasted 140 s. The sound source is nearest to the receiver at 62nd s. Figure 1 shows the spectrum change throughout the 140-s time span. At the start point of the 140 s, the moving object was far from the receiver, the difference between the direct path and the reflection path was not great. According to formula (3) or (6), it is known that the interference notches now have a big frequency separation of Δf_{dr}. The left side of Figure 1 shows that the gaps between the waves are wider. As the sound source came near the receiver, frequency separations between the interference notches narrowed, as illustrated in Figure 1, by narrower gaps between the waves. The narrowest gaps could be found at the 62nd s, indicating smallest frequency separation. With the sound source moving away from the receiver, Δf_{dr} increased and the gaps between the waves became wider. These results conform to the on site experiment that was conducted at sea.

3. CEPSTRUM ANALYSIS AND THE TIME DELAY DIFFERENCE MEASUREMENT

The next step is to find a way to measure the frequency separation between the interference notches in the normalized short-time spectrograph of the radiated noise of the sound source, and the time delay differences among different paths. This paper used the method of cepstrum analysis and succeeds in measuring the frequency separation between

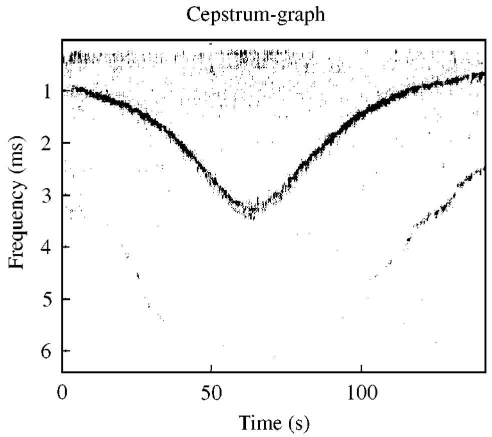

Figure 3. Cepstrum-graph of a vessel radiated noise.

the interference notches. If only the interference pattern of the direct path and the sea surface reflection path can be established, one δ line would be found in the cepstrum, which was produced with the term of $2H_1H_2\cos 2\pi f(\tau_r - \tau_d)$ in formula (2). This δ line is positioned at $(\tau_r - \tau_d)$ second in the cepstrum. If the interference between the direct path and the sea surface reflection path as well as that between the direct path and the sea bottom reflection path are expressed in the pattern, three δ lines, respectively, at $(\tau_r - \tau_d)$, $(\tau_r - \tau_d)$ and $(\tau_r - \tau_d)$ s should appear in the cepstrum in ideal circumstances.

If all the time-varying cepstra are put together with a different grayscale indicating the strength, then a cepstrum-graph is formed. Figure 3 is such a cepstrum-graph showing the radiated noise of the moving sound source which was used in the experiment. Since the strength of cepstrum is expressed with a different grayscale, the δ line in the graph is shown as a bright point (depicted in the figure as dark gray), which stretches into a bright line with the extension of time. The bell-shaped symmetric curve at the 62nd s is formed with the interference by the sea surface reflection path and the direct path. The two lines on each side are formed with the interference by the sea bottom reflection path and the direct path. The δ line produced by the interference of the sea bottom reflection and the surface reflection does not appear in the graph in our case.

4. ESTIMATION OF THE RANGE AND DEPTH BY USING TWO TIME DELAY DIFFERENCES

4.1. FORMULA DEDUCTION

In the above section, the time delay differences of the three paths are obtained through cepstrum analysis. This section tries to deduce the formula for the estimation of the range and depth by using two time delay differences τ_1 and τ_2 from three paths. Figure 2 shows the geometry. Using geometrical knowledge, the formula of range R_d and the sine of angle α can be found

$$R_d = \frac{D_3D_1(D_3 + D_1) - C^2(\tau_2^2D_1 + \tau_1^2D_3)}{2C(\tau_1D_3 - \tau_2D_1)}, \tag{9}$$

$$\sin\alpha = \frac{C(\tau_1D_3 + \tau_2D_1)}{2D_1D_3} + \frac{C^2(\tau_1^2D_3 - \tau_2^2D_1)}{4R_dD_1D_3} + \frac{(D_3 - D_1)}{4R_d}, \tag{10}$$

where $D_3 = F_2F_1$, $D_3 = F_2F_3$, the time delay difference τ_1 between the direct path and the surface reflection path is equal to $(R_r - R_d)/C$, the time delay difference τ_2 between the direct path and the bottom reflection path is equal to $(R_r' - R_d)/C$. $D_3 = 2H_r$, $D_1 = 2(H_{sea} - H_r)$, and H_{sea} is the depth of the sea. While F_2 is the surface mirror image of the receiver, F_3 is the sea bottom mirror image of the receiver. Therefore, both the horizontal and vertical distances to the sound source can be found as

$$H_h = R\cos\alpha, \qquad H_h = H_r - R_d \sin\alpha. \tag{11, 12}$$

4.2. ESTIMATION OF THE RANGE AND DEPTH OF THE MOVING SOUND SOURCE BY USING TWO TIME DELAY DIFFERENCES

The formula of the relations between the range and the time delay difference has been deduced in the above section. Using the moving sound source in the experiment as an example, this section will explain the way to estimate the range and depth.

In this experiment, the time delay difference between the direct path and the surface reflection path and that between the direct path and the bottom reflection path could both be measured only within a limited time span (i.e., 108th–140th s). The values are shown in Figure 4(a) and 4(b). Figure 4(a) is the time delay difference of the interference by the bottom

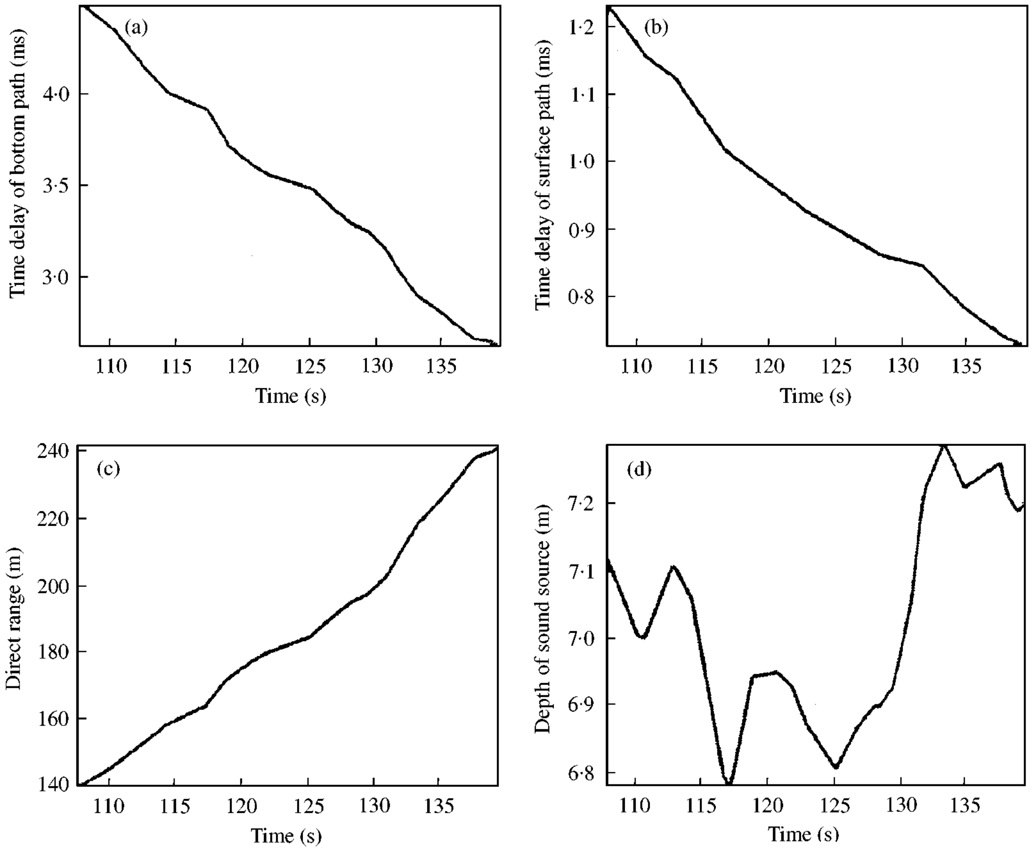

Figure 4. Estimation of range and depth by using two time delay differences of paths.

reflection path and the direct path, while Figure 4(b) is the time delay difference of the interference by the surface reflection path and the direct path. These difference values are drawn from Figure 3—the cepstrum-graph. If the surface and the bottom reflection time delay differences from 108th–140th s are substituted into formula (9), the range R_d to the sound source can be obtained, as expressed in Figure 4(c). If R_d is substituted into formula (10), the result would be the sine of range α. The horizontal and vertical distances to the moving sound source can be obtained by using formulas (11) and (12) respectively. The depth of the moving sound source is shown in Figure 4(d). In the process of calculation, it is known that the sea depth is 35 m, and the depth of the receiver in 18 m. Figure 4(c) gives the varying range of the moving sound source from 140 to 240 m in the time span of 105th–140th s. Figure 4(d) gives the depth of the moving sound source as about 7 m. The estimated range and depth conform to the data of the experiment on the sea.

5. ESTIMATION OF THE RANGE AND DEPTH BY USING ONE TIME DELAY DIFFERENCE

In the above section, the estimation of the range and depth by using two time delay differences, those of the bottom and surface reflection paths is discussed. However, in most time span of this experiment, only the time delay difference of the surface reflection and the direct reflection could be measured. This section tries to deduce the formula for estimating the range to the moving sound source by using a single time delay difference.

5.1. FORMULA DEDUCTION

Figure 2 shows the geometry of the sea surface reflection path and the direct path. SF_2 is the range of the surface reflection path R_r, and SF_1 is the range of the direct path. It is known that if the time delay difference of the surface reflection path and the direct reflection path is $\tau_1 = \tau_r - \tau_d$, then we have the formula:

$$\tau_1 = (R_r - R_d)/C \qquad \Delta R = R_r - R_d = C\tau_1 \tag{13, 14}$$

in which ΔR is the path difference. When F_1 and F_2 are expressed as two focuses on a hyperbola, and when it is known that $\Delta R = C\Delta\tau$, then the sound source should be a point on the hyperbola. Using the hyperbola property, we have the hyperbola equation

$$\frac{x^2}{(\Delta R/2)^2} - \frac{y^2}{H_r^2 - (\Delta R/2)^2} = 1, \tag{15}$$

in which axes x and y are as shown in Figure 2, while the co-ordinate of the sound source is $x = -H_t$, $y = H_h$. Supposing that the depth of the second source is known, the horizontal distance of the sound source to the receiver H_h can be found as follows:

$$H_h = \sqrt{\left(\frac{4H_t^2}{C^2\tau_1^2} - 1\right)\left(H_r^2 - \frac{C^2\tau_1^2}{4}\right)}. \tag{16}$$

Using formula (16) deduced in this section, the time delay difference of the surface reflection path and the direct path $\tau_r - \tau_d$ can be produced from the cepstrum graph and the horizontal distance to the moving sound source can as well be found. It would also be easy to find the direct range to the moving sound source:

$$R_d = \sqrt{H_h^2 + (H_r - H_t)^2}. \tag{17}$$

5.2. ESTIMATION OF THE RANGE OF THE MOVING SOUND SOURCE BY USING ONE TIME DELAY DIFFERENCE

The time delay difference of the surface reflection path and the direct path from 10th to 140th s can be found in Figure 3 cepstrum graph, as shown in Figure 5(a). It is known that the depth of the receiver is 18 m at the time of our experiment, and the depth of the sound source is 7 m as calculated in section 4.2, therefore, in formula (16), H_r is 18 m and H_t is 7 m. When substituted into formula (16), the horizontal distance to the moving sound source during this time span can be found. The direct range to the moving sound source can also be found with formula (17), as shown in Figure 5(b). From the above calculations, it is known that the moving sound source was about 160 m away from the receiver at the 10th s and then came nearer to the receiver. The nearest range was about 45 m at 62nd s and then it moved farther and farther away. It was 240 m away from the receiver at the 140th s. Regrettably, the paper can only provide the new approach of measuring the range and depth of a submerged moving sound source by using ceptrum analysis, but not the proven accuracy of such measuring method. The experiment was for purposes other than testing the method of measuring the range and depth. In the experiment, both the depth of the moving sound source and the depth of the receiving sensor were known. The sound source was moving in a straight line and its nearest point to the receiving sensor was about tens of meters away. Due to the limitations of the on-site experiment, more precise data were not available.

6. DISCUSSIONS AND CONCLUSIONS

In shallow waters, a short-time spectrograph of radiated noise from a moving sound source as shown in Figure 1, can often be collected. The time delay difference can be measured with cepstrum analysis. The range and depth of the moving sound source to the receiver can be found with the formulas deduced in this paper. The sea depth is assumed as known and the formulas are deduced by supposing that the sea bottom is flat. If two time

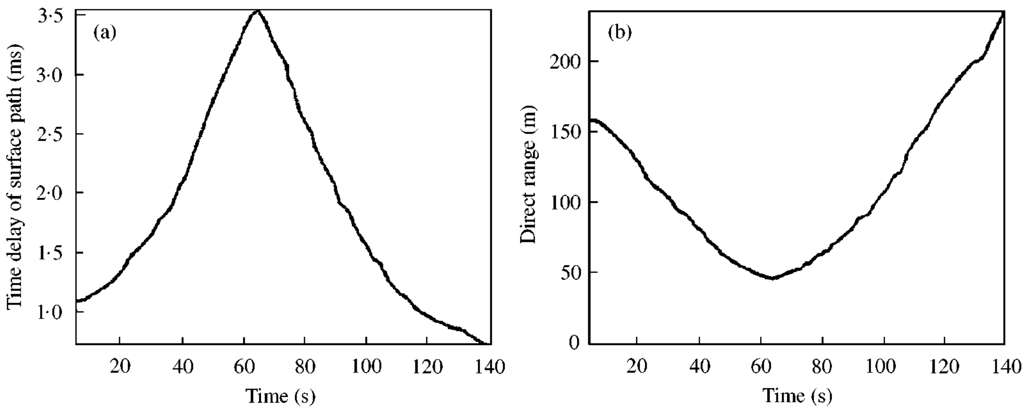

Figure 5. Range and depth estimate using one time delay difference.

delay differences can be found (that between the surface reflection and the direct paths and that between the bottom reflection and the direct paths), both the range and the depth of the moving sound source can be found. If there is only one time delay difference (that between the surface reflection and the direct paths), then the range can be found when the depth of the sound source is known. If the depth of the sound source is unknown, the range might be estimated by using the restraining conditions of the moving speed of the sound source.

REFERENCES

1. J. C. HASSAB 1974 *Journal of Sound and Vibration* **35**, 489–501. Time delay processing near the ocean surface.
2. J. C. GOOLD 1996 *Journal of the Acoustical Society of America* **100**, 3431–3441. Signal processing techniques for acoustic measurement of sperm whale body lengths.
3. S. K. MITCHELL, N. R. BEDFORD and M. S. WEINSTEIN 1976 *Journal of the Acoustical Society of America* **60**, 825–828. Determination of source depth from the spectra of small explosions observed at long ranges.
4. R. L. JOHNSON, Q. R. BLACK and A. G. SONSTEBY 1994 *IEEE Transactions on Aerospace and Electronic Systems* **30**, 462–470. HF multipath passive single site radio location.
5. J. S. BENDAT and A. G. PEIRSOL 1980 *Engineering Applications of Correlation and Spectral Analysis*. New York: A Wiley-Interscience publication, 142.